1. Theorem 1 – C,

We can rewrite the quotient into a product such as . Then we can simplify such using which is equivalent to . Therefore .

6. Can a sequence have infinitely many limit points? no limit point?

**Infinitely many limit points**

<1, 1, 2, 1, 2, 3, 1, 2, 3, 4, ...> as <***xi***> where ***xi*** =***n*** precisely when ***i*** = (n2+(2k−1)n+(k2−3k+2)) / 2 for some k ∈ N.

To justify this formula, we start by showing that ***n*** first appears in the sequence in the term with index n(n+1)/2. By splitting the sequence into (1), (1,2), (1,2,3), (1,2,3,4), ..., (1, ..., n), it is pretty clear that ***n***has index 1+2+3+4+...+n, which can be shown to equal n(n+1)/2 by induction.

Then we continue this useful grouping of the sequence to show the indices of the subsequent terms. (1), ..., (1, ..., n), (1, ..., n, n+1), (1, ..., n, n+1, n+2), ... The second appearance comes ***n*** terms after the first appearance, the third appearance comes (n+1) terms after the second, and so on. In general, the kth appearance of ***n*** will come n + (n+1) + ... + (n+k−2) terms after the first appearance. This can be easily shown to be (k−2)(k−1)/2 − (n−1)n/2.

But then the kth appearance of ***n*** in the sequence will have index n(n+1)/2 +(k−2)(k−1)/2 − (n−1)n/2, which simplifies to (n2+(2k−1)n+(k2−3k+2)) / 2.

Once you have defined this sequence, showing it has infinitely many limit points is easy. We say that ***m*** is a limit point of <***xi***> precisely if there is a subsequence of <***xi***> converging to ***m***. Using f(n,k) = (n2+(2k−1)n+(k2−3k+2)) / 2 as our choice function, we choose the subsequence <***yi***> where yi = xf(m,i)=m. It stands that <***yi***> converges to ***m***.

**Proof by contradiction**

Limit ***a*** exists considering the interval (***a*** – 1, ***a*** + 1). For all ***n*** sufficiently large, ***xn*** > ***a*** + 1

So ***xn ∉*** (***a*** – 1, ***a*** + 1). This contradicts the definition of a limit point.

11.

a.

When substituting, the first 5 elements of the sequence is { }. Using this, it is obvious that the absolute value approaches 1 which means that when

, elements will be between -1 and 1. Therefore, the **infimum** **is -2** and the **supremum is** .

b.

When substituting, the first 5 elements of the sequence is {}. Using this, we can see that there are no upper bounds. Therefore, there is **no supremum** in this sequence. When ***n*** is an arbitrary integer ***1/n*** is almost 0. The sequence also jumps between large values and 1 when ***n*** is even and odd, respectively. Therefore, the **infimum is 0**.

c.

no substitution is needed as we can see this is a ***sin*** function. ***Sin*** function can only be within -1 to 1 and the range of ***n*** is limited to the **supremum of** and the **infimum of** .

d.

When substituting, the first 5 elements of the sequence is { }. Using this, we can see that there are no negative elements. Therefore, the **infimum is 0.** When ***n*** is odd, the sequence becomes ***n2*** which has no upper bounds. Therefore, there is **no supremum**.

13 –b. Construct an example of bounded sequences { ***an*** } and { ***bn*** } where

A bounded sequence that represents the equation above can be true when and . Both are bounded with their **supremum as 1** and **infimum as -1**.

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